MATHEMATICAL EQUATIONS FOR THE SEAMS OF A BASEBALL IN 3D SPACE

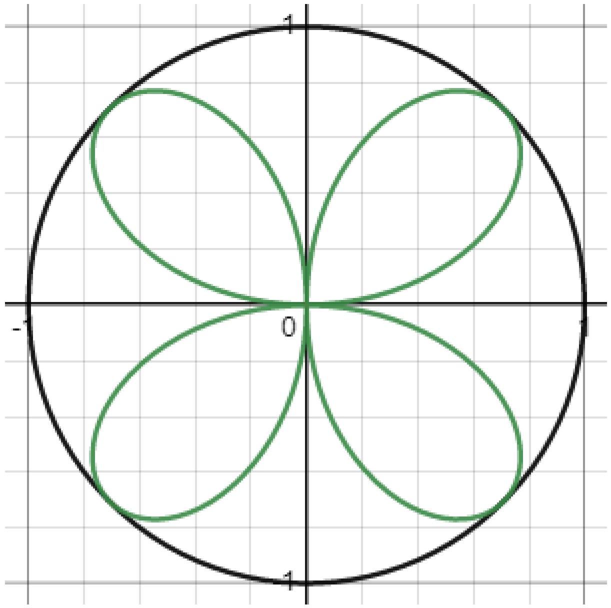
The purpose of this blog post is to explain how I came up with an equation that models the seams of a baseball in 3D space. This equation is subject to a scaling variable that allows for the distance of separation between the seams to be controlled. I then also give a preview of how we then can go from a 3D spherical map to a 2D one. Any information that will be explained at a later date will have a notation "future post n", where n will be the post number.

1. Create a sphere with radius 1

2. Create an equation for seams that lies exactly on the surface of the sphere
   a. I did not find an equation to my liking, so I made it
      i. x = $sign(\cos(2t)) \cdot \sqrt{1 - \left(\left((1-a)\cos(t)\sin(2t) + a\sin(t) \right)^2 + \left((1-a)\sin(t)\sin(2t) + a\cos(t) \right)^2 \right)}$
      ii. y = $(1-a)\cos(t)\sin(2t) + a\sin(t)$
      iii. z = $(1-a)\sin(t)\sin(2t) + a\cos(t)$

b. Ignoring x for the moment, we have y=(1-a)cos(t)sin(2t)+a*sin(t) and z=(1-a)sin(t)sin(2t)+a*cos(t)

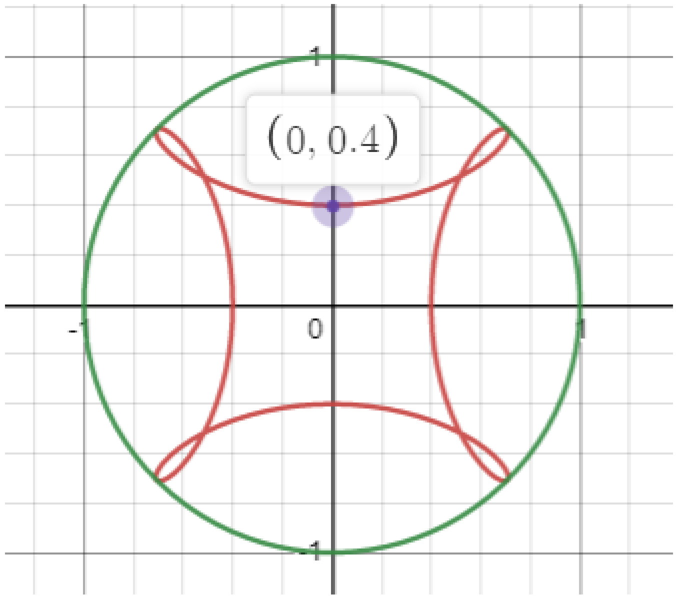
i. The graph for a 4 petal rose is r=sin(2θ) for 0<=θ<=2π

1. 

ii. Converting from polar to Cartesian coordinates with horizontal axis y and vertical axis z, we get y=cos(t)sin(2t) and z=sin(t)sin(2t) for 0<=t<=2π (using "t" here instead of "θ")

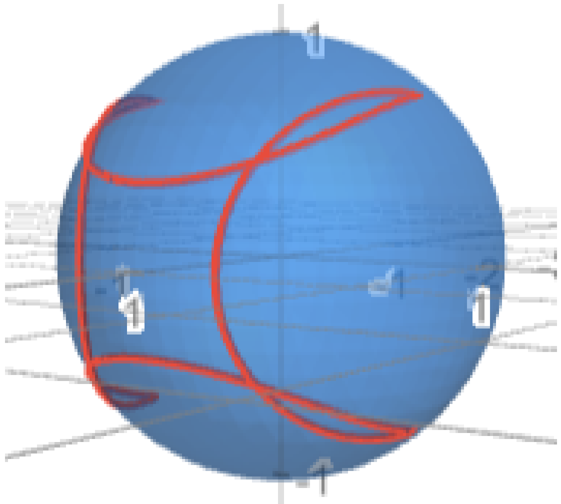
iii. However, the graph intersects the origin, and we need it to not do that. A forthcoming video (future post 1) will explain how we get from a 4-petal rose to projected seams, but for the moment you can see that it involves pushing the lines that intersect the origin to now be farther away. This distance is represented by the value "a".

c. Plotting this at a=0.4 (this specific value for a will be explained in future post 2) and t ranging from 0 to 2π makes this shape:

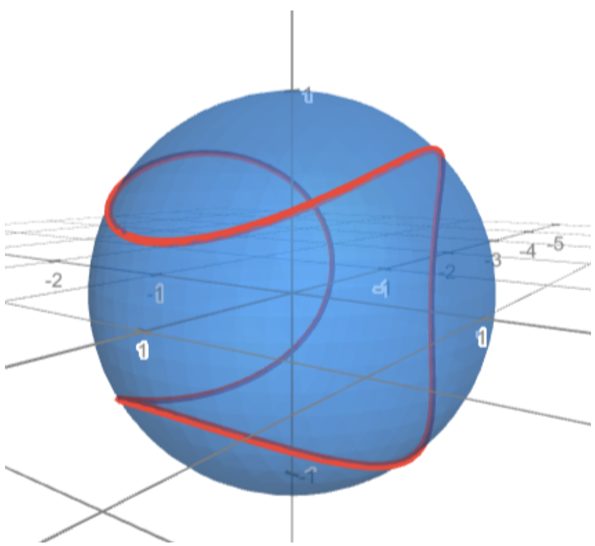
i. 

d. This is as if the entire ball was squished onto the plane x=0 and you could only see the seams and the edge of the ball

e. We know $x^2+y^2+z^2=1$ because this is a sphere of radius 1. We just set y and z. Solve for x. Unfortunately, the square root makes this shape:

   i. 

f. To get the seams to be separated properly, find a multiplier on the square root. Using testing to which values of t result in the correct separation (which will be elaborated on in future post 1), it can be confirmed that “sign(cos(2t))” is the correct multiplier. Plotting the entire parametric we get this:

   i. 

3. 3D space can be described either using 3 spatial dimensions or using 2 rotational dimensions and a radial dimension that all originate from the point (0,0,0). Since the sphere is of radius 1, the radial dimension is a constant. This in effect reduces the variables from 3 to 2. The 2 rotational dimensions we will use are latitude and longitude, like the Earth.

    a. We are assuming that
        i. 3D point (1,0,0) is the surface point (0 long, 0 lat)
        ii. Commissioner's signature is at (0 long, 0 lat), specifically the letter U in "MAJOR LEAGUE BASEBALL"

    b. The vector in 3D space that describes a point on a sphere of radius 1 using these assumptions is
        i. $\left[\cos(T)\cdot\cos(p), \sin(T)\cdot\cos(p), \sin(p)\right]$
            1. This will be explained in future post 2
        ii. T is that longitude number and p is the latitude number, converted into the proper system
            1. Check values: tan(45)=1 for degrees or tan(π/4)=1 for radians

4. We now have
    a. a curve in 3D space that describes the seams on the surface of a sphere in terms of some "time" variable t
        i. Note that a curve is just a series of points, where each point corresponds to a different value for t

    b. a vector that describes a point on the surface of a sphere in terms of latitude and longitude

5. These are 2 different ways to represent a point on the surface of a sphere. If we wanted to find the latitude and longitude of points on the seams, we can set the two point equations equal and solve for latitude and longitude in terms of the “time” variable. To save you the trouble, here is the finished set of equations (which will be explained in future post 3):

$$f(t) = (1-a)\sin(t)\sin(2t) + a\cos(t)$$

$$g(t) = (1-a)\cos(t)\sin(2t) + a\sin(t)$$

$$A(t) = \arcsin\left(f(t)\right)$$

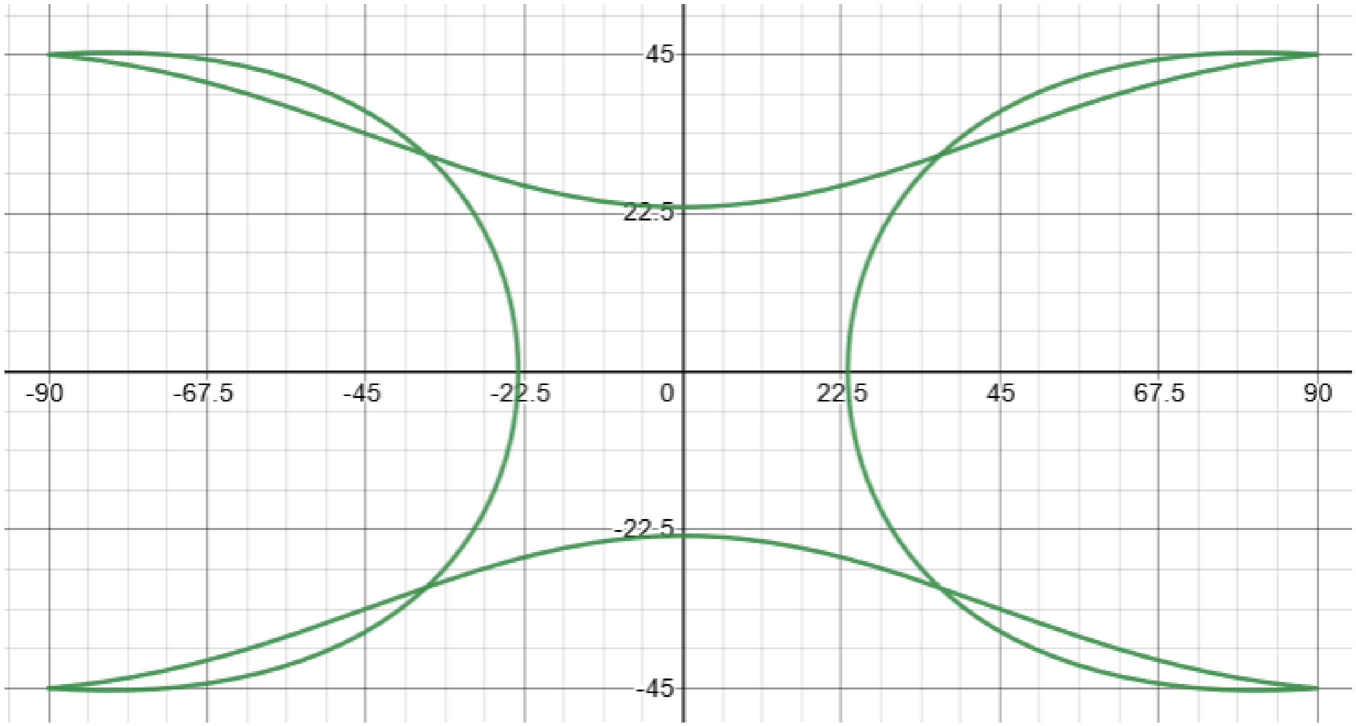
$$O(t) = \arcsin\left(\frac{g(t)}{\sqrt{1-\left(f(t)\right)^2}}\right)$$

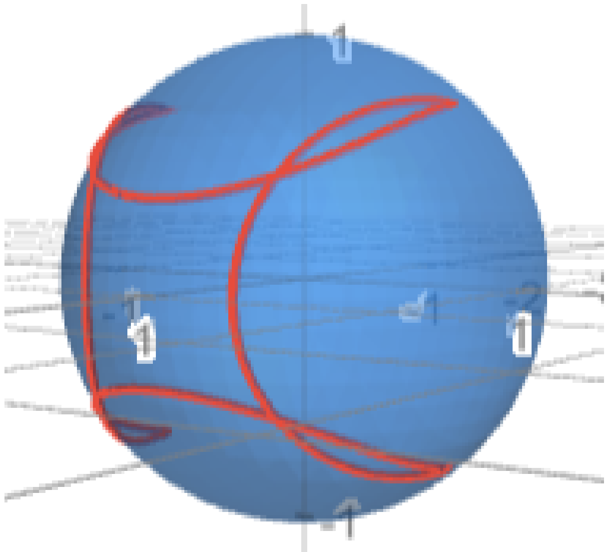
$$\left(\frac{180}{\pi}O(t), \frac{180}{\pi}A(t)\right)$$

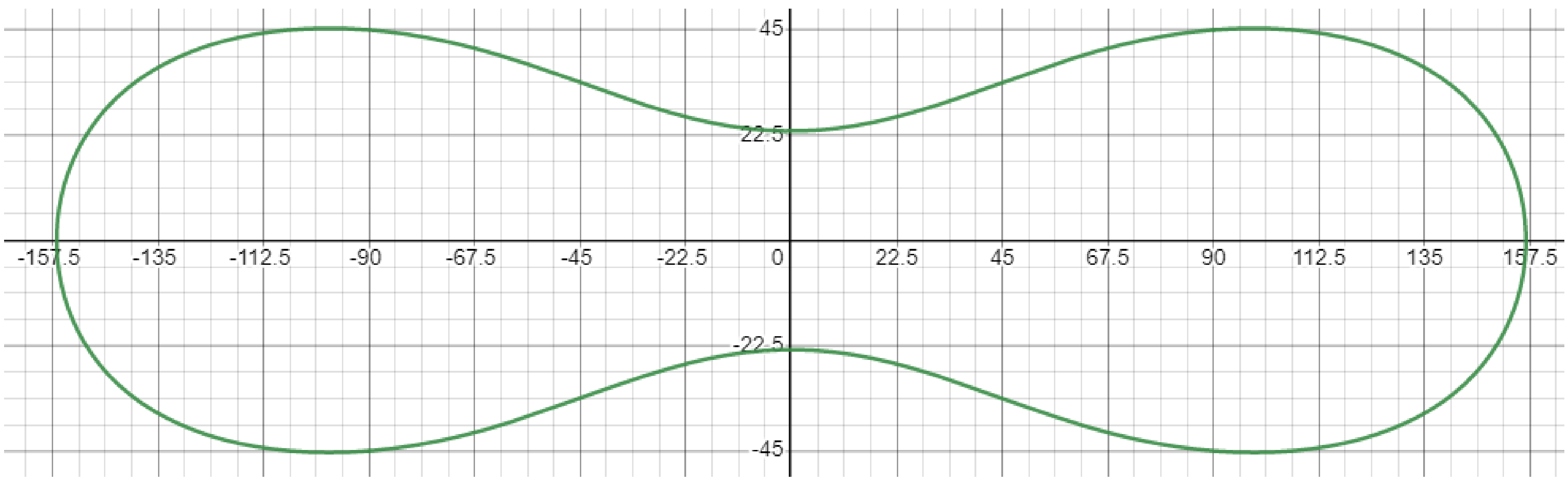
$$\frac{\pi}{4} \leq t \leq \frac{\pi}{4} + 2\pi$$

a.
   i. Note that the t variable is in radians. However, the final result of ((180/π)*O(t),(180/π)*A(t)) displays the angle in degrees. Do not mess with these parameters. I tried to and it messed up the plots.

6. The plot of ((180/π)*O(t),(180/π)*A(t)) looks like this:

a. 

7. However, something is wrong here. We know that the longitude for the seams should go around from -180 to 180. So why does it stop at 90? Well, remember this?
   a. 

8. The equation conversion is not suited to handle inverse functions for angles. Arcsin outputs values from -90 degrees to 90 degrees, so getting anything in a 360 range is not a one-step process. All we need to do is reflect the inward curving parts over the ±90 lines and the longitude-latitude plot is fixed!
   a. 

9. Future post 4 will explain how we can now use this graph to compare to different seam orientations, and the type of mapping that we chose to properly represent the spreads given a perspective issue that arises from using the mapping presented in this post.